Godel numbering.

$$p \xmapsto{\text{inj}} n \in \mathbb{N}_0$$

For a prop.

$\exists$ a unique Godel number

Ernest Nagal
& James. Newman
1958

$1 = S0$

$9 = \underbrace{S \cdots S.0}_{9}$

Constant sign	Gödel number	Usual Meaning
¬	1	not
∨	2	or
⊃ ⇒	3	if...then...
∃ ·	4	there is an...
=	5	equals
0	6	zero
S·	7	the successor of
(	8	punctuation mark
)	9	punctuation mark
,	10	punctuation mark
+	11	plus
×	12	times
x	13	var
y	17	var
z	19	var

Any proposition in an effective Peano-arithmetic system can be written in the above. Letters in linear order.

$$0 = 0 \longmapsto \underset{P_1}{2^5} \times \underset{P_2}{3^6} \times \underset{P_3}{5^5}$$
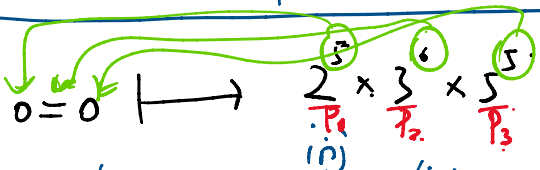
Recall:

$$\mathbb{N}_0 \times \mathbb{N}_0 \xrightarrow{(\cap)} \mathbb{N}_0$$
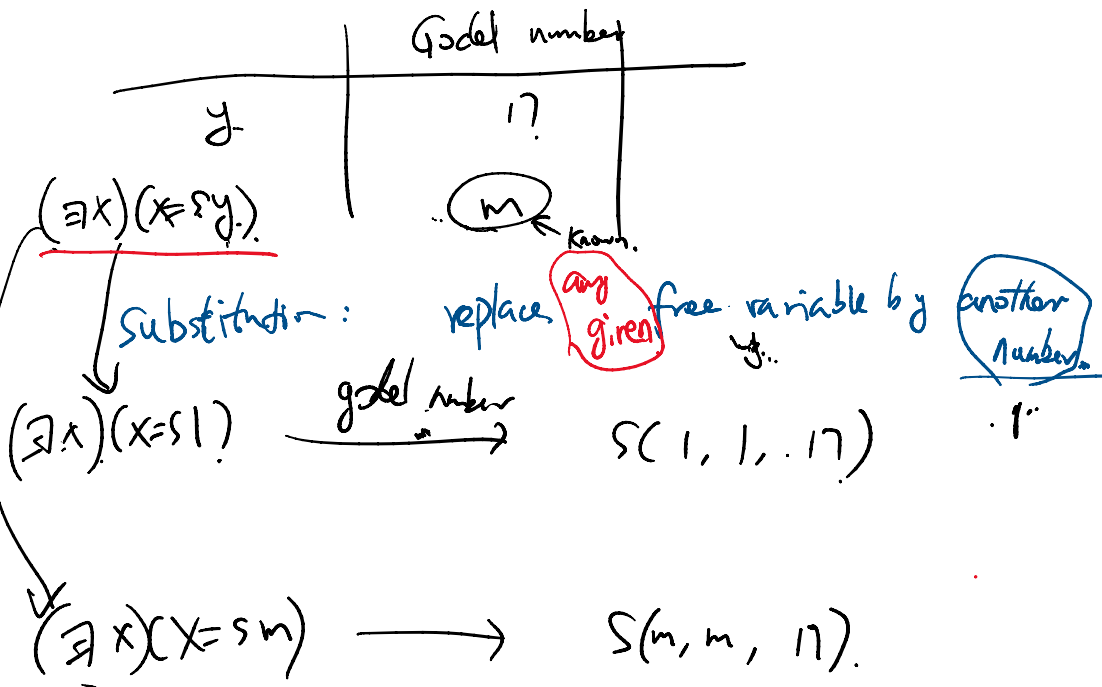
$$(a, b) \longmapsto 2^a \times 3^b$$

Rk ① : Godel number is not a surjective map.

② : If. p has Godel number n.
    then we say $\underline{A(n, p) = 1}$.

If $A(n, p) = 1$. then from n,
by unique prime factorization of n,
we know p..

— free variable

$(\exists \textcircled{x}) (x = s \, \underline{y})$ → free variable

all other are free.

→ not free

"behind quantifiers"

"y has a successor"

	Gödel number
y	17

$(\exists x)(x = sy).$

$(\exists x)(x = s1)$ → gödel number → $S(1, 1, .17)$

$(\exists x)(x = sm)$ → $S(m, m, 17).$

replace any given free variable by another number

$\textcircled{m}$ ← known.

y..

.1.

"The formula $sub(y, y, 17)$ cannot be proved"

with gödel number.

Gödel number. n

"The formula $sub(n, n, 17)$ cannot be proved"

with Gödel number

G statement.

G refers to its own Gödel

G statement .

Sa refers in its own model
number

Sub(a, n, 17).