$[n+1] = \{1, 2, \dots, n+1\}$

$[n] = \{1, \dots, n\}$

$\{1,2\}$ $\{1,2,3\}$

$\{1\}$

The formula with Godel number $sub(y, y, 17)$ is not provable

Recall: A^{fixed} is a formula.

$n = \phi\left((\exists x)(x = Sy)\right)$ free

$$\boxed{A(n, p) = \begin{cases} 1 & \text{if } \phi(p) = n \\ 0 & \text{otherwise} \end{cases}}$$

$\phi(A) = n \longleftrightarrow$ Godel number. (prime factoring)

$sub(m, (\ ) =$

$\phi((\exists x)(x = Sy))$

$\underline{n}$: $\underline{\text{numeral}}$, encoding of a formula

$\frac{n!}{2!\,2}\ n$ $\binom{n+1}{2} \cdot n(n-1)!$

$x, b \in \mathbb{N}_0$

$$proof(n, m) = \begin{cases} 1, & \text{if } n = \phi(\text{proof of } p) \text{ s.t. } \phi(p) = m \\ 0 & \text{otherwise} \end{cases}$$

"$\underline{n}$ is a proof of $\underline{m}$"

$a_{n+2} = \frac{3a_{n+1}}{3(3a_n - 2a_n)}$

$\begin{pmatrix} 7 & -6 \\ 3 & -2 \end{pmatrix}$

$\begin{pmatrix} a_{n+2} \\ a_{n+1} \end{pmatrix}$

$\underline{THM}(b) := (\exists x)\left(proof(x, b)\right)$ "b has a proof"

$\exists\, x$ s.t. $x = \phi(\text{proof of } p)$ and $\phi(p) = b$.

Want refer this to itself.

$\neg THM(b)$ "b is not provable"

Define a function $sub(m, n) = j$ if

$-2a_n = 7a_n - 6a_n$

$m = \phi(A(x))$ where x is the only free variable.

$j = \phi(A[n])$ $[n]$ numeral of n.

a_n

a_{n+1}

$sub(m, n)$ does:

compute m to get $A(x)$

1. take n as a number.
2. Do $A([n])$
3. Compute $j = \phi(A([n]))$
4.

upshot: $sub(m, n)$ is computable over $\mathbb{N}_0$; it is an $\underline{\text{arithmetic function}}$.

itself has a formula in the formal system.

Godel has to define $\underline{SUB(x, y, z)}$ as an explicit formula

$[x]\ [y]\ [z]$

Consider $A(x) := \neg THM(sub(x, x))$

$\phi(A(x)) = \underline{N}$

$G = A([N])$ is not provable & true.

because

If G is not true by $A(x)$ is provable

If G is provable, by $A(x)$, it is not provable