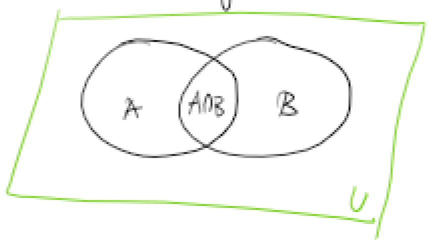
Elementary set theory

set

A B

Venn diagram



What is a set?

A collection of objects

backpack → set.

pen. book. box → elements

Capital letter A, B, Q. to denote a set

lower case letter → elements

1000 elements

{ . }

finite set: $\{1, 2, 3\}$

infinite set: $\{\infty \text{ elements}\}$

$\{(x, y) \mid y = x^2\}$ → descriptive

$\in \mathbb{R}$

what kind of object?

such that

S.t.

conditions

same → two sets have identically elements

$\{1, 2\} = \{2, 1\}$ elements are unordered

$\{1, 2\} \overset{\text{same}}{=} \{1, 2, 2\}$ repeated elements count one.

$\underbrace{0.\dot{9}9\cdots9}_{\infty} \overset{\text{same}}{=} 1$

$\underbrace{0.9\cdots9}_{1000}$ 1.

$(0.9\cdots9)$ $(1.0\cdots0)$

elements

Empty set $\quad \emptyset \neq \{0\}$ → single element set of 0

$\downarrow \quad \cancel{=} \quad \{\emptyset\}$ → .... $\emptyset$

$\{\ \}$

Number systems

$$\boxed{\mathbb{N} = \{1, 2, 3 \dots\}} \rightarrow \text{natural}$$

$\{0, 1, 2, 3 \dots\}$ → non-negative integers

$\mathbb{Z} = \{\dots -3, -2, -1, 0, 1, 2, 3 \dots\}$ → integers

Solve $qx = p$

$x = \frac{p}{q}$

$\mathbb{Q} = \left\{ \frac{p}{q} \;\middle|\; p, q \text{ are integers}, q \neq 0 \right\}$ → rationals

$\cap$

e, π → π is not rational $\quad$ Euler

$\boxed{e^{\pi} \text{ rational ?}}$ $\qquad$ $\boxed{e + \pi \text{ rational ?}}$

$\mathbb{A} = \left\{ a \in \mathbb{C} \;\middle|\; a \text{ is the solution for some } \sum_{i=0}^{n} c_i x^i = 0, \ c_i \in \mathbb{Q} \right\}$

↓ algebraic numbers

$c_n x^n + c_{n-1} x^{n-1} + \dots c_1 x + c_0 = 0$

$\mathbb{R} \xleftrightarrow{1-1}$ line → real

$\mathbb{C}$: extend $\mathbb{R}$ by $\sqrt{-1}$. → complex

$\updownarrow^{1-1}$ plane

Operations on sets

① $x \in A$ or $x \notin A$

② subset $\underline{A \subseteq B}$ if any $x \in A$ then $x \in B$.

same

supset $B \supseteq A$

proper subset $A \subsetneqq B$: $A \subseteq B$
there is some $x \in B$ s.t. $x \notin A$.

① ϕ is a subset of any set.

② subset of $A = \{\{1, 2\}, 3, 4\}$

$\{1, 2\}$ no!

$\{\{1, 2\}\}$ yes.

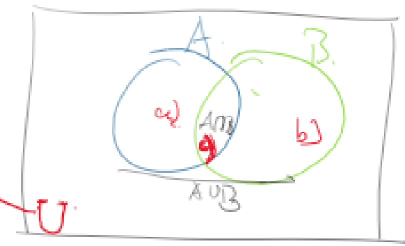
$\{3, 4\}$ yes.

$\{1, 2, 3\}$ no.

intersection and union.

universe

universe of discourse



$A\cup B = \{x \mid x\in A \text{ or } x\in B\}$

$A\cap B = \{x \mid x\in A \text{ and } x\in B\}$

a) $x\in A,\ x\notin B$
b) $x\notin A,\ x\in B$
c) $x\in A$ and $x\in A$

$A\cup B\cup C \cdots\cdots \cup L_{100}$ finite union

$A\cap \cdots\cdots \cap L_{1000}$ finite intersection

algebraic

$\sum_{i=1}^{N} x_i$ ✓

$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ ✓

$\sum_{n=1}^{\infty} \frac{1}{n} = \infty$ ✓

add up all real numbers between 0 and 1 → illegal.

$$\bigcup_{i=1}^{\infty} A_i = A_1\cup A_2\cup\cdots\cup A_\infty$$

$$\bigcup_{x\in[0,1]} B_x(\varepsilon)$$

legal → well defined

$B(\varepsilon) \rightarrow$ p istrue

ZFC system

Cantor

Choice

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surprisingly useful fact:

prove $A = B$

we need $A \subseteq B$ and $B \subseteq A$.

Complements:

complement of A (in U)

$$A^c = \left\{ x \;\middle|\; \begin{array}{l} x \in U \\ \text{and } x \notin A \end{array} \right\}$$

$U =$ plane
$A =$ disk

$A^c =$ 

$U = \mathbb{R}$ $A = [0,1]$

$A^c = (-\infty, 0) \cup (1, \infty)$

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set minus (difference)

$$\underline{A-B} \;\; \underline{A\setminus B} := A\cap B^c := \{x \mid \overset{x\in U}{x\in A \text{ and } x\notin B}\}$$

e.g. $U=\mathbb{R}$ $\quad A=[0,1]$

$B=(\frac{1}{2},\infty)$

$A\setminus B=[0,\frac{1}{2}]$

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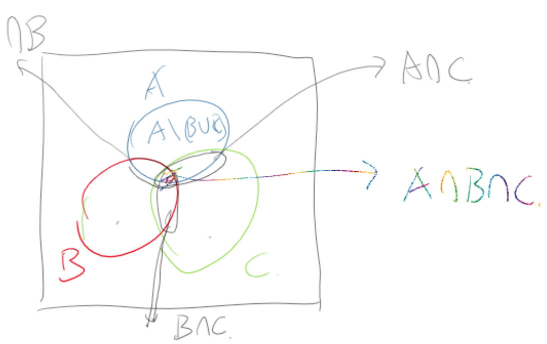
complements & set difference works for ~~finitely~~ many sets
countably

A, B, C



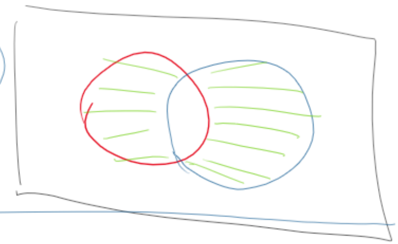
?!

$$\boxed{\mathbb{R}\setminus\bigcup_{x\in\mathbb{R}}\{x\} = \emptyset \qquad \bigcap_{x\in\mathbb{R}} \mathbb{R}\setminus\{x\}}$$

symmetric difference

$$A \triangle B = (A-B) \cup (B-A)$$

$$= (A \cup B) \setminus (A \cap B)$$



Cardinality, ordering of sets

$|A|$ or $card(A)$ # of elements in set A

when A is a finite set

$|A|$ is just counting.

$$|A \cup B| = |A| + |B| - |A \cap B|$$

When A is infinite, want $|A|$ still preserve the above law.

Power set $P(S)$ of S

2^S

$S = \{a, b, c\}$

$P(S) = \{ \phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, S \}$

$S \xrightarrow{f} \{0, 1\}$

$f(x) = 0 \quad x \notin$ subset

$f(x) = 1. \quad x \in$ subset.

$\# f = |P(S)|$

$\Rightarrow \quad |P(S)| = 2^{|S|}$ when $|S| = \infty$

want this to be true.

$\aleph_0 < ? < 2^{\aleph_0}$ $\quad \boxed{2^{\infty}} \overset{?}{=} \infty$

Cantor.

ordered pairs

set elements do not have order.

$(a, b) \in \mathbb{R}^2$

Cartesian product $\quad A \times B$ (set, set)

$= \{(a, b) \mid a \in A,\ b \in B\}$

$\boxed{|A \times B| = |A| \times |B|}$

$A_1 \times \cdots \times A_n$ or $\prod_{i \in \mathbb{N}} A_i$

disjoint union $\quad \{1_A, 2_A\} \amalg \{1_B, 3_B\}$

$= \{1_A, 1_B, 2_A, 3_B\}$

Math | CS

multiset? ⟵ List ⟵ [1, 2, 2]

Set ⟵ *set([1, 2, 2]) = {1, 2}

$\boxed{\text{vector}}$ ⟵ array = (1, 2, 2, ...)

an element in Cartesian product.