Mathematical Logic

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$1 \neq 2$ or $1 < 2$

0th-order logic. ← <u>propositional</u> logic

1st-order logic ← $\boxed{\text{predicate}}$ logic

2nd-order $\underline{y} > 2$

⋮ ⋮

nth-order logic.

→ "formal logic"

→ law of identity "A is A"

→ law of non-contradiction $\boxed{A}$ cannot be both true and false

→ law of excluded middle

either true or false.

<u>Def</u>: statement or proposition. p, q

"The statement is false" is not a statement.

Truth value: $T = 1$

$F = 0$

negation "$\neg p$" not p.

Truth table.

$p \mid \neg p$

negation $\neg p$" not p.

p	$\neg p$
T	F
F	T

$$p = (1+1=2)$$

$$\neg p = (1+2 \neq 2)$$

double negation $\longrightarrow$ original statement

$$p = \neg(\neg p)$$

Compound statements, conjunctions & Disjunctions

and — or

p, q — $p \wedge q$ — $p \vee q$

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

different from natural language.

p and $\neg q$
q and $\neg p$
p and q

De Morgan's law.

$$\boxed{\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q \qquad (A \cup B)^c = A^c \cap B^c}$$

$$p \Rightarrow q$$

imply.

$$\neg(p \wedge q) \Leftrightarrow \neg p$$

$$\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$$

$$(A \cap B)^c = A^c \cup B^c$$

Conditional statements

If .... , then ....

$p \Rightarrow q$

p is a sufficient condition of q.

q is a necessary condition of p.

p only if q.

negate: $p \Rightarrow q$

contrapositive

$\neg q \Rightarrow \neg p$

$\neg(\neg q \Rightarrow \neg p)$

$\neg p \vee q$

$\neg(p \Rightarrow q)$

$\Updownarrow$

$p \wedge \neg q$

predicates

"$y > 2$"

$P(y)$

$\neg P(y) == y \le 2$.

quantifiers

$\exists\, y \in \mathbb{R},\ P(y)$ — there exist a y, s.t. $y > 2$

$\forall\, y \in \mathbb{R},\ P(y)$ — for all y real, $y > 2$.

$\forall$ $\leftarrow$ (Every integer is even.

$\exists$ $\leftarrow$ (There are even integers.

Fermat's Last Theorem:

$$\forall n \in \mathbb{N}, \left(\exists\, a, b, c \in \mathbb{N},\ a^n + b^n = c^n \right) \Rightarrow (n \leq 2)$$

Math Induction principle

H/S

$$\left[S(1) \wedge \left(\underset{\forall n \in \mathbb{N},}{S(n)} \Rightarrow S(n+1) \right) \right] \Rightarrow \left(\forall m \in \mathbb{N},\ S(m) \right).$$

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negation & quantifier.

$\neg$ (Every integer is even)

We have at least one integer, which is not even.

$\neg$ (there is a even number) $=$ We cannot find any even integer.

$\nexists$

$$\neg(\forall x, p(x)) \Longleftrightarrow \exists y \; \neg p(y)$$

$$\neg(\exists w, p(w)) \Longleftrightarrow \forall v, \neg p(v).$$