Math 433 W 2019 - Suggested HW for curvatures

Not for turn in, but for preparation of Exam 2, solutions are posted but strongly encourage to do it before look at the solution. Note that Exam 2 is on Apr 3rd, 11:00(sharp)-11:50, which covers 2-5, 2-6, 3-2, 3-3, 4-2, 4-3 in do Carmo's book. Most questions will be about 1st, 2nd fundamental form and (Gaussian, principal, normal) curvatures.

Lesson plan, reading assignments and homework:

Week 10-11: Sections 4-3,2-4 in do Carmo's book, 1st step to global geometry.

This HW contains 5 problems from do Carmo's book, and is aimed to preparing for the Exam 2.

We strongly recommend everybody to try each problem by oneself before looking at the solution.

1.. What is the Gaussian curvature of a surface of revolution?

Hint: Use the "good" parametrization we gave in class.

2. Show that the mean curvature H at $p \in S$ is given by

$$H = \frac{1}{\pi} \int_0^\pi \kappa_n(\theta) d\theta,$$

where $\kappa_n(\theta)$ is the normal curvature at p along a direction making an angle θ with a fixed direction.

3. If the surface S_1 and S_2 intersect along a regular curve C, then the curvature k of C at p is given by

$$k^2 \sin^2\theta = \lambda_1^2 + \lambda_2^2 - 2\lambda_1\lambda_2 \cos\theta,$$

where λ_1 and λ_2 are the normal curvatures at p, along the tangent line to C, of S_1 and S_2, respectively, and θ is the angle made up by the normal vectors of S_1 and S_2 at p.

4. Local existence of isothemal coordinates. Here isothemal coordinates means $I = \lambda(u,v)du^2 + \lambda(u,v)dv^2$. This means every surface is locally conformal.